Evolutionary Game Theory and Economic Applications

Math 250 – Game Theory

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This study explores the relationship between the Hawk-Dove model derived from evolutionary game theory and its applications in microeconomics and macroeconomics. Several hypothetical examples are explained in detail.

An introduction

I began this project researching Evolution and Selection through the lens of Game Theory. I read through John Maynard Smith's book, *Evolution and the Theory of Games*, along with watching several lectures provided by Yale on the topic. These were my main sources, supported by several other explanations of the concepts presented in the book. After reflecting for some time over a project topic, the idea was given that the concepts presented in Evolutionary Game Theory are more interdisciplinary than I originally thought. In other words, the concepts which I was learning and understanding could be applied across a wide spectrum of situations such Political Science and Economics.

The idea of approaching entrepreneurship from a background of Evolutionary Game Theory seemed appealing and is directly related to projects that I am currently working on in other fields regarding my own business, Studio Ace of Spade. This fueled my fire and I began to explore the possibility of applying game theory to my own business' situation through the lens of evolution.

To understand more specifically how Evolutionary Game Theory can be applied to entrepreneurship, it is necessary to understand the game theory underlying certain elements of evolution.

What is Evolutionary Game Theory?

Evolutionary Game Theory is the application of game theory concepts to situations in which a population exists with a set of strategy choices and is dependent upon the interaction and evolution of the population. A main factor of evolutionary game theory is that anything a player does depends heavily upon what other players do. Evolutionary game theory in this sense is frequency-dependent. A frequency-dependent game is centered on that concept that if a player decides to act a certain way in a given environment, success will be dependent upon the frequency with which the player meets weaker or stronger strategies. Players can evolve, reproduce, and learn. Thus, opponents can also evolve. A player's best move will always depend upon the strategy of the opponent.

Another main idea of evolutionary game theory is that evolutionary game theory is co-evolutionary. Players evolve together throughout the course of the game. Again, this plays on the idea of interactivity over a span of time. The way that players evolve is through choice of strategy and the amount of players with a given strategy inside of a population. It should be stressed that these strategies are only valid within that given population. Populations will not compete or evolve against one another as this model is inadequate for that type of evolutionary study and analysis.

Another important concept that will play a strong role throughout this study is the idea of an Evolutionary Stable Strategy, or an ESS. An ESS can be thought of under its more commonly known name, the Nash Equilibrium with an added condition. An ESS is a strategy that, once fixed in a population resists invasion through natural selection alone. If a strategy exists en masse in a population and it can prevent another strategy from entering the game over an extended period of time, it is considered stable. Thus an ESS must have the property that if almost all members of the population adopt it, then the fitness of the members will be greater than that of any possible mutant. If that happened to be false, then a mutant could invade the population and the strategy would not be an ESS.

There is also the case of the mixed ESS which I will be exploring later on. This comes about when players are not able to adopt mixed strategies as individuals. The model then compensates this with a mixed amount of each pure strategy in the population and that balanced nature prevents any invaders from entering the game.

The Hawk-Dove Model

The Hawk-Dove model is a more simplistic model used in evolutionary game theory. It defines a frequency-dependent game that considers pairwise contests between players. More complex models can be approached by adding more players, but for the context of this study, we will only examine situations in which there are pairwise contests.

The situation is this: Two animals, or players, are contesting a resource of value V. When a player obtains V, the overall Darwinian fitness of the winner is increased by value V. Animals can use one of two strategies – Hawk or Dove. The hawk strategy will escalate the situation and will continue to do so until injured or the opponent retreats from the situation. Hawks will not take into account anything about the other players including size, fitness, strength, etc. If the hawk is injured, the player's Darwinian fitness is reduced by cost C. Since this game will be repeated many times over, we will assume that 50% of the time a hawk will win over another hawk. The dove strategy is to retreat if encountering a hawk or to share the resource of value V if encountering a dove. The dove then divides V evenly between the two players. The last assumption that we will make is that players reproduce asexually in amounts equal to their Darwinian fitness levels.

To make this a little more precise, we will need to define a few functions. Firstly, W(H) will be the notation for the fitness of the hawk. W(D) then will be the logical representation for the fitness of the dove. E(H,D) represents the payoff to an individual adopting a hawk strategy against a dove opponent. Logically, E(D,H), E(D,D), and E(H,H) would be the other notations for the payoffs received in their respective situations. This payoff matrix can then be abstracted from this information.

Hawk-Dove Payoff Matrix (Row Player, Col Player)	*Hawk*	*Dove*
Hawk	E(H,H) = (.5(V-C), .5(V-C))	E(H,D) = (V, 0)
Dove	E(D,H) = (0, V)	E(D,D) = (V/2, V/2)

Table 1 – Generalized Hawk-Dove Payoff Matrix

Now, we will define two random strategies which are X and Y. With our definition of a stable strategy, X will be an ESS as long as `W(X) > W(Y)`. In simpler terms, if the fitness of strategy X is greater than the fitness of strategy Y, strategy X is an ESS. Now, we will assume that Y is a mutant attempting to invade a population of X. Normally, mutations will occur at very low frequencies when first invading a population. Then, if that is the case, one of the following two options must be true:

1. `E(X,X) > E(X,Y)`
2. `E(X,X) = E(Y,X)` and `E(X,Y) > E(Y,Y)`

Based upon the previous example using strategies X and Y, it is clear that dove is not an ESS. Simply put, it can be seen that dove meets neither of the previous two conditions. E(D,D) can clearly never be greater than E(D,H). Even if V were 0, the payoffs would be equal. By the same notion, it can be seen that hawk is in fact an ESS so long as `V > C`. This is logical as if C happened to be greater than V, then the cost of fighting would reduce fitness even for the winner. If

$V < C$, the cost of injury is high relative to the reward of victory, then we are going to find a mixed strategy that will be an ESS. For example, when doves enter a hawk population and $V < C$, it actually is beneficial for the doves as they can run from encounters with Hawks at no cost while Hawks are damaging each other severely while contesting resources.

We can generalize this more using a payoff matrix as such:

Hawk-Dove Payoff Matrix (Row Player, Col Player)	*Hawk*	*Dove*
Hawk	A	B
Dove	C	D

Table 2 – Abstract Dove Payoff Matrix

We can now assume that if $A > C$, then the hawk strategy is an ESS. By the same logic, if $B > D$, then the dove strategy is an ESS. If neither of these are true, then there must be a mixed strategy ESS. This can be solved for using the Bishop-Cannings theorem which states $(1-P)A + pB = (1-P)C + pD$ which can always be solved if $A < C$ and $B < D$. Therfore, the possibilities for finding an ESS are this:

1. Hawk is an ESS
2. Dove is an ESS
3. There is a mixed strategy ESS

John Maynard Smith, in his book, *Evolution and the Theory* of Games, shows that the third option – that there is a mixed strategy solution – will always be stable. The proof follows:

> *To show that this solution is stable, consider the alternative strategy* $q=q(H)+(1-q)(D)$*. Since the strategy* $I=P(H)+(1-P)(D)$ *has the property that* $E(H,I)=E(D,I)$*, it follows that* $E(q,I)=E(I,I)$*. Hence, I will be stable if* $E(I,q)>E(q,q)$*. Now:*
>
> $$E(I,q)-E(q,q) = E(I,q)-E(I,I)+E(q,I)-E(q,q)$$

$$=(p'-q')V(q-p)$$

$$=(p-q)^2\ (b+c-a-d)$$

Since C > A and B > D and q does not equal p, it follows that E(I,q) > E(q,q) and hence, I is stable.

How does this apply to Entrepreneurship?

There are two main ways which we could apply the Hawk-Dove game in a business setting – macroeconomically and microeconomically. We will first approach the game from a macroeconomic standpoint.

We define a situation in which a business can be either a hawk or a dove for a market segment on a national scale. The population will be the total number of businesses in the market segment. The cost C will be an investment for finding work, such as advertising expenses. The value V will be the revenue from the work completed. Businesses will enter and leave the market based upon the success and fitness which they achieve from their chosen strategy. The businesses "reproduce" in the sense that new businesses entering the market segment will wish to emulate businesses which they assume are successful based upon their fitness rating. New businesses are created with the same strategy as their "parent". We can assume that any mutations are businesses who believe that they have a good idea and wish to go against the norm. In this particular case, players cannot adopt a mixed strategy, but can only use a pure strategy. Thus, the model would have a mixed ESS, not simply an ESS.

The second way to apply this model in a business situation that I will explore is in a microeconomic setting. We define a situation in which there is competition between two businesses, A and B. The population in this particular case is going to be the target market or market segment in which these two businesses compete. The market segment is unable to compete against other market segments and the businesses are limited to working only within the defined market segment. The strategy names are changed from hawk and dove to invest and inquire, respectively. Investing refers to the idea that businesses must invest money into a specific project to win the job and receive the payoff. An inquire strategy simply means that a company inquires on a job and invests nothing into it. If an inquire comes up against an investment, it loses and the invest strategy wins the job. If two inquiries come up on a given job, the companies decide to split the job down the middle and work on it collaboratively. Our cost C is now defined as a loss of profit (or fitness) and value V is a gain in revenue. The payoff that a company receives represents an addition or subtraction to its fitness, which is how many jobs the business can take on in the following quarter. Businesses will choose a given strategy with which to pursue a job.

Thus, when we look at this through entrepreneurial eyes, we see that we have now defined a situation in which the Hawk-Dove game models a game between two businesses competing for market share. It also will lead us to the same logical conclusion that being a hawk when the revenue from the job is greater than the cost investing into will be an ESS.

Hypothetical and specific examples of business applications

Now, we will create two hypothetical situations in order to demonstrate the Hawk-Dove model's application in business and entrepreneurship. For the purposes of this paper, I will limit my explorations to simplistic, pairwise models. In the following example, I will be exploring situations in which value is less than cost; specifically, I am interested in analyzing situations in which cost to enter into the market is high and not necessarily beneficial to directly compete with other business all of the time. Firstly, we will approach the macroeconomic model.

We need to define a market segment. In this case, we will approach a hypothetical market for manufacturing companies who produce RV parts. The cost C will be representative of total costs of aggressively finding work such as the costs of advertising and pitching ideas. The value V will represent the average value of an order placed to any company for RV parts. The invest strategy will refer to a company whose business model tells them to aggressively pursue a job, and the inquire strategy will refer to a company who does not aggressively pursue work. As mentioned before, invest is the equivalent of the hawk strategy from the original game and inquire is equivalent to the dove strategy.

The strategies will interact as such:

1. *Invest v. Invest:* Both companies continue to spend money trying to get a job until one company receives the work. Both must pay cost C regardless of whether or not they are the winner. If a company wins, its fitness increases by value V.
2. *Invest v. Inquire:* The inquiring company allows the investing company to have the job and backs off as it will not spend any money to get the job. The investing company receives the full value V.
3. *Inquire v. Invest:* This works the same way as the previous situation with the investing company receiving value V.
4. *Inquire v. Inquire:* Two companies inquire about a job and the work is split down the middle. Thus, each company receives value V/2.

Lastly, we need to make assumptions about what the average cost of aggressively getting a job is. In this case, we will assume it is $60,000, while the value of completing a job is $50,000. The payoff matrix then appears as such:

Hawk-Dove Payoff Matrix (Row, Col) in 1000's	*Invest*	*Inquiry*
Invest	E(H,H) = (-5, -5)	E(H,D) = (50, 0)
Inquire	E(D,H) = (0, 50)	E(D,D) = (25, 25)

Table 3 – Macroeconomic Invest-Inquire Payoff Matrix (values in 1000's)

Since $V < C$ in this case, invest cannot be an ESS. Inquire is not an ESS either based upon previous arguments. Thus, a mixed strategy will be necessary to find a mixed ESS as players can only have pure strategies. We know that a mixed strategy ESS exists simply as the Nash Equilibrium Theorem states that every strategic game has at least one Nash equilibrium in pure or mixed form. Since there is no pure strategy best response, a mixed strategy equilibrium must exist.

In order to find the Nash equilibrium, both invest and inquire must be best responses. Thus, we must create a situation where there will be indifference between the two choices. We can define a strategy such that $(1-p)\text{invest} + p(\text{inquire})$ creates indifference between the two choices. Therefore, if we set the two strategies equal to each other and substitute in the payoffs for all choices, we get:

$$
\begin{aligned}
-5(1-p) + 50p &= 0(1-p) + 25p \\
-5 + 55p &= 25p \\
-5 &= -30p \\
p &= 1/6
\end{aligned}
$$

This tells us that the market will be comprised of 1/6 of businesses whose strategy is inquire. Logically, $1 - 1/6 = 5/6$, and the remaining 5/6 of the market belongs to businesses who have adopted the invest strategy.

This also allows for us to determine the expected payoff for each game played, regardless of whether the business is using the invest strategy or the inquire strategy. We can do this by using our previous equation, `(1-p)invest + p(inquire)` and substituting in our payoffs.

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-5(1-1/6) + 50(1/6) = expected payoff
     -4.166 + 8.333 = e.p.
               e.p. = 4.166
```

Therefore, our expected payoff is $4,166 per game played in this market, regardless of what strategy the business is using.

Now, let us apply this in a microeconomic setting. We define two businesses, Business A and Business B. These two businesses are in a particular market segment in which there is no other competition. They compete for a share of the market segment which is comprised of jobs. The market segment, then, is the population and each game played represents the competition between the two businesses in the web design and development field for a particular job. We will keep the invest and inquire strategies from the macroeconomic game, as well as how they interact. The main difference is that each business has a choice of the strategy they choose to acquire a given job. The cost C will represent the money (or fitness) invested on a particular job in order to acquire it, and value V refers to the money (or fitness) gained from acquiring and completing a job. We will assume that the cost to acquire a job is $8,000 and the value received from a job, on average, is $6,000. We then arrive at this payoff matrix:

Hawk-Dove Payoff Matrix (Row, Col) in 1000's	*Invest*	*Inquire*
Invest	E(H,H) = (-1, -1)	E(H,D) = (6, 0)
Inquire	E(D,H) = (0, 6)	E(D,D) = (3, 3)

Table 4 – Microeconomic Invest-Inquire Payoff Matrix (values in 1000's)

As in the previous macroeconomic problem, $V < C$ in this case and the invest strategy cannot be an ESS. Inquire is not an ESS either based upon previous arguments. Therefore, we are again looking for a mixed strategy which will be a Nash Equilibrium. This requires that both the invest strategy and the inquire strategy must be best responses. We will again create a situation where there will be indifference between the two strategy choices. Inputting our payoffs into the equation gives us:

$$-1(1-p) + 6p = 0(1-p) + 3p$$
$$-1 + 7p = 3p$$
$$-1 = -4p$$
$$p = 1/4$$

Using our previous logic, this means that business should be using the inquire strategy 25% of the time. Therefore, it also means that the remaining 75% of situations should be adopting the invest strategy. Using the same logic as before, we are able to determine the expected payoff for each completed job in this market by inserting the value of p into our previous equations as such:

$$-1(1-1/4) + 6(1/4) = \text{expected payoff}$$
$$-.75 + 1.5 = \text{e.p.}$$
$$\text{e.p.} = .75$$

Therefore, our expected payoff is $750 per game played in this market.

Shortcomings of the Hawk-Dove Business Model

In the previous hypothetical situations, the work was done under very constricting assumptions. There were several key issues with these models. One of those issues was that the macroeconomic model assumed that the businesses in the game couldn't adopt a mixed strategy of their own. This is a problem as virtually no business would ever constantly compete using such a strict strategy. Something else that is assumed is that there is not emigration or immigration into the population from surrounding environments. A large influx of outside competitors would be enough to upset an ESS or mixed ESS enough to cause an invasion.

Lastly, it assumes that the environment will remain constant. Costs and values change over time, job availability could plummet, etc. With the two hypothetical situations we investigated, we essentially investigated evolving populations and stability over the course of an extended period of time that occurs within a “snapshot” of an environment.

Summation and Possibilities of Future Work

As we’ve seen, these models provide interesting possibilities for analyzing situations. However, they cannot be used to make solid decisions in the real world simply because they limit the possibilities too greatly.

I would have liked to explore more complex models during the course of this study, but the scope of the mathematics that would have been used to analyze intricate situations was beyond my understanding of game theory at this point in time.

This study did provide a more thorough understanding of business interactions and market competitions, though. If I were able to continue working on this study beyond the simplistic models that I’ve analyzed here, I would begin working on situations in which there are more than two players competing for a resource. I would also like to further investigate the War of Attrition model, which basically alters the payoffs of this game such that when two doves encounter each other, the payoff is zero.

Acknowledgements

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